in the discussion of Arnoldi's eigenvalue method where the authors use "Arnoldi leminscates" to illustrate the convergence of the method.

The authors also stress the interrelation between algorithms. For example they use a four-way division of Krylov sequence methods (linear systems vs. eigenproblems and Hermitian vs. non-Hermitian) to guide their discussion. Again, the authors make an amusing distinction between "orthogonal structuring" and "structured orthogonalization" to illustrate the difference between algorithms based on Householder transformations and those based on orthogonalizing a sequence of vectors.

The book concludes with an essay by Trefethen on the definition of numerical analysis. One does not have to agree with the definition itself to appreciate the important issues Trefethen raises so entertainingly.

I have two reservations about the book—neither damning. First, there could be more stress on implementation issues (e.g., convergence criteria for the QR algorithm). It is natural that a book of this sort would not spend a great deal of time on the minutiae, but given the many ways you can shoot yourself in the foot while computing with matrices, a few more examples of the pitfalls would be helpful. Second, the material and presentation was developed for graduate students at two high-powered institutions (MIT and Cornell). I would certainly not say that the book is unsuitable for other schools, but the instructor who uses it should be prepared to field some difficult questions.

These reservations aside, I can strongly recommend this book. The authors are to be congratulated on producing a fresh and lively introduction to a fundamental area of numerical analysis.

G. W. Stewart

2[65-01, 65Lxx, 65Mxx]—*A first course in the numerical analysis of differential equations*, by Arieh Iserles, Cambridge Texts in Applied Mathematics, Cambridge University Press, New York, New York, 1996, xvi+378 pp., softcover, \$27.95, hardcover, \$74.95

This is a lively textbook that is suited for mathematics graduate students or for well-prepared (mathematically) engineering students. This text is, on the one hand, rigorous and concise in its presentation of mathematical ideas and, on the other hand, verbose in its discussion of the big picture, i.e., "the ways and means whereby computational algorithms are implemented" and developed. To quote Professor Iserles: "In this volume we strive to steer a middle course between the complementary vices of mathematical nitpicking and of hand-waving".

This monograph is devoted to the numerical analysis of both ordinary and partial differential equations but, as needed, many other traditional topics are introduced and studied. These include interpolatory quadrature, Newton's method (and its variants) in $\mathbf{R}^d$, Gaussian elimination, iterative methods for sparse linear systems, and the FFT. There are several appendices, called "*Bluffer's guide to useful mathematics*", wherein important definitions and theorems from linear algebra, approximation theory, and ordinary differential equations are presented. Each chapter concludes with a short but challenging list of exercises and a Comments and Bibliography section. The text is organized into three parts. Part I consists of six chapters devoted to numerical methods for ODE's. Part II deals with elliptic PDE's and Part III with evolution equations.

The material in Part I is as follows.

1. *Euler's method and beyond.* This introductory chapter is concerned with simple single step methods for first-order nonlinear systems of standard form, $y'(t) = f(t, y)$, with Lipschitz right-hand side. The presentation is limited to Euler's method, the implicit Trapezoidal rule, and the so-called θ-method that includes each of the aforementioned schemes as special cases. As an indication of the level of exposition, we mention that the implicit function theorem in $\mathbf{R}^d$ is invoked on p. 14 in the error analysis of the θ-method. 2. *Multistep methods.* This chapter is devoted to the Adams family of multistep methods, both explicit and implicit, and to backward difference formulae. There is a careful analysis of order and stability via the root condition. The classical Dahlquist convergence theorem is given and discussed. 3. *Runge-Kutta methods.* The chapter begins with a discussion of interpolatory quadrature with emphasis on Gaussian rules. Then explicit (ERK) and implicit (IRK) methods of Runge-Kutta type are developed. The chapter concludes with a derivation of collocation schemes that result in IRK methods. In the Comments and Bibliography section there is an introduction to the graph theoretic derivation of RK methods (due to Butcher). 4. *Stiff equations.* To begin the chapter, Professor Iserles has given an extensive and revealing discussion of a simple stiff system having one stable and one unstable eigenvector. In discussing the temptation to increase stepsize after the unstable eigenvector component has decayed, he admonishes the reader with the delightful analogy: "like a malign version of the Chesire cat, the rogue eigenvector might seem to have disappeared, but its hideous grin stays and is bound to thwart our endeavors". The bulk of the chapter is devoted to issues of A-stability for Runge-Kutta and multistep methods. 5. *Error control.* Up to this point in the text the author has not been concerned with the practical implementation of the many numerical methods derived and analyzed in previous chapters. The estimation of local errors (using another method in tandem) and controlling the error with stepsize changes is the theme of this chapter. A particularly nice aspect of the chapter is Iserles' device of applying each of the error-control devices to three specific simple systems of ODE's, one of which is moderately stiff. The presentation is limited to halving or doubling of the stepsize; however, both multistep and single step methods are considered. 6. *Nonlinear algebraic systems.* This final chapter of Part I is concerned with Newton's method (and variants) as applied to the solution of those nonlinear systems that arise in implicit methods (both RK and multistep) for ODE's. The Banach fixed point theorem is proved and utilized to establish convergence of the methods considered. The issue of *starting* the iteration is addressed in the last section wherein predictor-corrector schemes (PECE) are discussed and the equally important issue of *stopping* the iteration is also presented.

There are six chapters devoted to elliptic PDE's and related matters in Part II.

7. *Finite difference schemes.* The famous 5-point difference approximation to the Laplacian is used to solve the Poisson problem on a rectangle. The eigenvalues of this discrete Laplacian are determined and shown to approximate the eigenvalues of the Laplace operator. The 9-point operator and a powerful modification thereof are derived and utilized on a model problem. 8. *The finite element method.* Much of this chapter is introductory in nature and intended to give the reader some feeling for the main ideas involved in the FEM. A two-point boundary problem is utilized as

a vehicle for the description of the FEM that is presented as a Galerkin method for the differential equation as well as a Ritz method for the minimization of the appropriate functional. More general self-adjoint elliptic problems and the corresponding FEM are described later in the chapter with careful statements of important tools (for existence of the FEM solution and error analysis) such as Cea's lemma and the Lax-Milgram theorem. 9. *Gaussian elimination for sparse linear equations.* This brief chapter examines the issue of "fill in" in Cholesky factorization and the use of graphs to investigate the sparsity structure and factorization of matrices. 10. *Iterative methods for sparse linear equations.* The classical Jacobi, Gauss-Seidel, and SOR methods are analyzed with emphasis on SOR. Unfortunately, the powerful conjugate gradient method is relegated to the Comments and Bibliography sections at the end of the chapter. 11. *Multigrid techniques.* The author motivates the multigrid technique by demonstrating the smoothing property of Gauss-Seidel thereby revealing how one may accelerate via a hierarchy of grids. Then the basic ideas of the V-cycle and full multigrid iteration are discussed. No error analysis is presented. 12. *Fast Poisson solvers.* This chapter is concerned with the use of FFT techniques to efficiently solve block Toeplitz, symmetric tridiagonal systems that arise in certain finite difference (element) approximations.

The final two chapters constitute Part III, namely partial differential equations of evolution.

13. *The diffusion equation.* The analysis, stability and convergence, of semidiscrete and fully discrete schemes for parabolic initial-boundary value problems is presented. The discussion is, by and large, limited to Euler and Crank-Nicolson time discretizations. 14. *Hyperbolic equations.* Professor Iserles motivates the development of numerical schemes for hyperbolic problems by considering the advection equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ and its numerical solution by Euler and Crank-Nicolson with particular attention to stability. The remainder of the chapter deals with the wave equation and Burgers equation. (On p. 308, Burgers is incorrectly stated; the expression $\frac{\partial u^2}{\partial x^2}$ should read $\frac{\partial u^2}{\partial x}$.)

This is a well-written, challenging introductory text that addresses the essential issues in the development of effective numerical schemes for the solution of differential equations: stability, convergence, and efficiency. The softcover edition is a terrific buy—I highly recommend it.

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3[65M06, 65M12, 65M20]—*Numerical methods for the three dimensional shallow water equations on supercomputers*, by E. D. de Goede, CWI Tract, Vol. 88, Stichting Mathematisch Centrum, Amsterdam, 1993, x+124 pp., 24 cm, softcover, Dfl. 40.00

This book is a collection of articles on the development of a numerical method for the three dimensional shallow water equations. They are obtained by simplifying the Navier-Stokes model: the unknowns are the horizontal velocity and the water elevation as for the two dimensional model, but the velocity may depend on the vertical coordinate. The pressure gradient is directly linked to the water elevation,